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Concepts of Graph Theory and is Applications

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Abstract: *The field of mathematics plays very important role in different fields. One of the important areas in mathematics is graph theory which is used in structural models. These structural preparations of various objects or technologies direct to new inventions and modifications in the existing environment for development in those fields. The field graph theory started its journey from the problem of Konigsberg Bridge in 1735. This paper gives an overview of the applications of graph theory in various fields to some extent but mainly focuses on the computer discipline applications that uses graph theoretical concepts. This paper gives an overview of the applications of graph theory in heterogeneous fields to some extent, but mainly focuses on the computer science applications and chemistry that uses graph theoretical concepts. Various papers based on graph theory have been studied related to scheduling concepts, computer science applications and an overview has been presented here.*

Keywords: Graphs, connectivity, constraints, graph coloring, graph drawing

I. Introduction

In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pair wise relations between objects. A graph in this context is made up of vertices which are connected by edges (also called links or lines). A distinction is made between undirected graphs, where edges link two vertices symmetrically, and directed graphs, where edges link two vertices asymmetrically. Graphs are one of the principal objects of study in discrete mathematics. Graph theory is nothing but a branch of Discrete Mathematics. Graph Theory is the study of graphs which are mathematical structures not only used in computer science but in many fields. Two problem areas are mainly considered. Problems such as classical problems and problems from an application. The classic problem is defined with the help of graph theory as connectivity, cut, path and flow, colorizing problems and theoretical aspects of graph drawing. The problems from an application focus on experimental research and implementation of graphs theory algorithms. Graphs are important because graph is a way of expressing information in pictorial form. A graph shows information that equivalent to many words. Many problems that are considered difficult to determine or implement can easily be solved by graphic theory. There are many types of graphs as part of graph theory. Each type of graph is related to a particular property. One of these graphs is used in many applications for troubleshooting. Because of the representation power of graphs and flexibility many problem can be represented and solved easily.

II. GRAPHS THEORY

Graphs provide a convenient way to represent various kinds of mathematical objects. Essentially, any graph is made up of two sets: 1- A set of vertices 2- A set of edges. Depending on the particular situation, restrictions are imposed on the type of edges we allow. For some problems directed edges are applied and for other problem undirected edges are applied from one vertex to other. So graphs give us many techniques and flexibility while defining and solving a real life problem.

Graphs has many features, some of them are:

- Provides abstracted view
- Establishes relationship among objects
- Balancing
- Modeling

- Decision -making ability
- Structural arrangement of various objects
- Easy modification or change in the existing system

III. HISTORY OF GRAPH THEORY

The origin of graph theory started with the problem of Koinsber Bridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koinsberg Bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem. Then in 1856, Thomas. P. Kirkman and William R.Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H.Dudeneymentioned a puzzle problem. Eventhough the four color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of Graph Theory.

IV. REPRESENTATION

A graph is an abstraction of relationships that emerge in nature; hence, it cannot be coupled to a certain representation. The way it is represented depends on the degree of convenience such representation provides for a certain application. The most common representations are the visual, in which, usually, vertices are drawn and connected by edges, and the tabular, in which rows of a table provide information about the relationships between the vertices within the graph.

Visual: Graph Drawing

Graphs are usually represented visually by drawing a point or circle for every vertex, and drawing a line between two vertices if they are connected by an edge. If the graph is directed, the direction is indicated by drawing an arrow. If the graph is weighted, the weight is added on the arrow.

A graph drawing should not be confused with the graph itself (the abstract, non-visual structure) as there are several ways to structure the graph drawing. All that matters is which vertices are connected to which others by how many edges and not the exact layout. In practice, it is often difficult to decide if two drawings represent the same graph. Depending on the problem domain some layouts may be better suited and easier to understand than others.

The pioneering work of W. T. Tutte was very influential on the subject of graph drawing. Among other achievements, he introduced the use of linear algebraic methods to obtain graph drawings.

Graph drawing also can be said to encompass problems that deal with the crossing number and its various generalizations. The crossing number of a graph is the minimum number of intersections between edges that a drawing of the graph in the plane must contain. For a planar graph, the crossing number is zero by definition. Drawings on surfaces other than the plane are also studied.

There are other techniques to visualize a graph away from vertices and edges, including circle packing's, intersection graph, and other visualizations of the adjacency matrix.

Tabular: Graph Data Structures

The tabular representation lends itself well to computational applications. There are different ways to store graphs in a computer system. The data structure used depends on both the graph structure and the algorithm used for manipulating the graph. Theoretically one can distinguish between list and matrix structures but in concrete applications the best structure is often a combination of both. List structures are often preferred for sparse graphs as they have smaller memory requirements. Matrix structures on the other hand provide faster access for some applications but can consume huge amounts of memory. Implementations of sparse matrix structures that are efficient on modern parallel computer architectures are an object of current investigation.

List structures include the edge list, an array of pairs of vertices, and the adjacency list, which separately lists the neighbors of each vertex: Much like the edge list, each vertex has a list of which vertices it is adjacent to.

Matrix structures include the incidence matrix, a matrix of 0's and 1's whose rows represent vertices and whose columns represent edges, and the adjacency matrix, in which both the rows and columns are indexed by vertices. In both cases a 1 indicates two adjacent objects and a 0 indicates two non-adjacent objects. The degree matrix indicates the degree of vertices. The Laplacian matrix is a modified form of the adjacency matrix that incorporates information about the degrees of the vertices, and is useful in some calculations such as Kirchhoff's theorem on the number of spanning trees of a graph. The distance matrix, like the adjacency matrix, has both its rows and columns indexed by vertices, but rather than containing a 0 or a 1 in each cell it contains the length of a shortest path between two vertices.

V. APPLICATIONS OF GRAPH THEORY

Graphs can be used to model many types of relations and processes in physical, biological, social and information systems. Many practical problems can be represented by graphs. Emphasizing their application to real-world systems, the term network is sometimes defined to mean a graph in which attributes (e.g. names) are associated with the vertices and edges, and the subject that expresses and understands real-world systems as a network is called network science.

Graph theory is playing an increasingly important role in the field of computer science. Any software that has to be developed, any program that has to be tested is making themselves easy using graphs. Its importance is derived from the fact that flow of control and flow of data for any program can be expressed in terms of directed graphs. Graph theory is also used in microchip designing, circuitry, scheduling problems in operating system, and data flow control between networks to networks. The theory of graphs had made the field of computers to develop its own graph theoretical algorithms. These algorithms are used in formulating solutions to many of computer science applications. Some algorithms are as follows:

- Shortest path algorithm in a network
- Kruskal's - minimum spanning tree
- Euler's- graph planarity
- Algorithms to find adjacency matrices.
- Algorithms to find the connectedness
- Algorithms to fi nd the cycles in a graph
- Algorithms for searching an element in a data structure (DFS, BFS) and so on.

A. Computer Science

The branch of computer science known as data structures uses graphs to represent networks of communication, data organization, computational devices, the flow of computation, etc. For instance, the link structure of a website can be represented by a directed graph, in which the vertices represent web pages and directed edges represent links from one page to another. A similar approach can be taken to problems in social media, travel, biology, computer chip design, mapping the progression of neuro-degenerative diseases, and many other fields. The development of algorithms to handle graphs is therefore of major interest in computer science. The transformation of graphs is often formalized and represented by graph rewrite systems. Complementary to graph transformation systems focusing on rule-based in-memory manipulation of graphs are graph databases geared towards transaction-safe, persistent storing and querying of graph-structured data.

B. Linguistics

Graph-theoretic methods, in various forms, have proven particularly useful in linguistics, since natural language often lends itself well to discrete structure. Traditionally, syntax and compositional semantics follow tree-based structures, whose expressive power lies in the principle of compositionality, modeled in a hierarchical graph. More contemporary approaches such as head-driven phrase structure grammar model the syntax of natural language using typed feature structures, which are directed acyclic graphs. Within lexical semantics, especially as applied to computers, modeling word meaning is easier when a given word is understood in terms of related words; semantic networks are therefore important in computational linguistics. Still, other methods in phonology (e.g. optimality theory, which uses lattice graphs) and morphology (e.g. finite-state morphology, using finite-state transducers) are common in the analysis of

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language as a graph. Indeed, the usefulness of this area of mathematics to linguistics has borne organizations such as TextGraphs, as well as various 'Net' projects, such as WordNet, VerbNet, and others.

C. Physics and Chemistry

Graph theory is also used to study molecules in chemistry and physics. In condensed matter physics, the three-dimensional structure of complicated simulated atomic structures can be studied quantitatively by gathering statistics on graph-theoretic properties related to the topology of the atoms. Also, "the Feynman graphs and rules of calculation summarize quantum field theory in a form in close contact with the experimental numbers one wants to understand." In chemistry a graph makes a natural model for a molecule, where vertices represent atoms and edges bonds. This approach is especially used in computer processing of molecular structures, ranging from chemical editors to database searching. In statistical physics, graphs can represent local connections between interacting parts of a system, as well as the dynamics of a physical process on such systems. Similarly, in computational neuroscience graphs can be used to represent functional connections between brain areas that interact to give rise to various cognitive processes, where the vertices represent different areas of the brain and the edges represent the connections between those areas. Graph theory plays an important role in electrical modeling of electrical networks, here, weights are associated with resistance of the wire segments to obtain electrical properties of network structures. Graphs are also used to represent the micro-scale channels of porous media, in which the vertices represent the pores and the edges represent the smaller channels connecting the pores. Chemical graph theory uses the molecular graph as a means to model molecules. Graphs and networks are excellent models to study and understand phase transitions and critical phenomena. Removal of nodes or edges leads to a critical transition where the network breaks into small clusters which are studied as a phase transition. This breakdown is studied via percolation theory.

D. Social Sciences

Graph theory is also widely used in sociology as a way, for example, to measure actors' prestige or to explore rumor spreading, notably through the use of social network analysis software. Under the umbrella of social networks are many different types of graphs. Acquaintanceship and friendship graphs describe whether people know each other. Influence graphs model whether certain people can influence the behavior of others. Finally, collaboration graphs model whether two people work together in a particular way, such as acting in a movie together.

E. Biology

Likewise, graph theory is useful in biology and conservation efforts where a vertex can represent regions where certain species exist (or inhabit) and the edges represent migration paths or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites or how changes to the movement can affect other species.

Graphs are also commonly used in molecular biology and genomics to model and analyse datasets with complex relationships. For example, graph-based methods are often used to 'cluster' cells together into cell-types in single-cell transcriptome analysis. Another use is to model genes or proteins in a pathway and study the relationships between them, such as metabolic pathways and gene regulatory networks. Evolutionary trees, ecological networks, and hierarchical clustering of gene expression patterns are also represented as graph structures.

Graph theory is also used in connectomics; nervous systems can be seen as a graph, where the nodes are neurons and the edges are the connections between them.

F. Mathematics

In mathematics, graphs are useful in geometry and certain parts of topology such as knot theory. Algebraic graph theory has close links with group theory. Algebraic graph theory has been applied to many areas including dynamic systems and complexity.

G. Others

A graph structure can be extended by assigning a weight to each edge of the graph. Graphs with weights, or weighted graphs, are used to represent structures in which pair wise connections have some numerical values. For example, if a graph represents a road network, the weights could represent the length of each road. There may be several weights associated with each edge, including distance (as in the previous example), travel time, or monetary cost. Such weighted graphs are commonly used to program GPS's, and travel-planning search engines that compare flight times and costs.

VI. CONCLUSION

The main objective of this paper is to present the importance of graph theory in different branches of science and our everyday life. This paper is valuable for students and researchers to get the overview of graph theory and its application in diverse fields like everyday life, computer science, Operation Research, Chemistry.

The main aim of this paper is to present the importance of graph theoretical ideas in various areas of compute applications for researches that they can use graph theoretical concepts for the research. An overview is presented especially to project the idea of graph theory. So, the graph theory section of each paper is given importance than to the other sections. Researches may get some information related to graph theory and its applications in computer field and can get some ideas related to their field of research. In this manuscript we reviewed applications of graph theory and research challenges have also been surveyed. The topics we reviewed include biochemistry, chemistry, communication networks, coding theory, algorithms, computation, operations research, x-ray crystallography, radar, astronomy, circuit design, communication network addressing and data base management. There are many problems in this area which are yet to be solved. It is hoped that this review would attract many new investigators into graph theory.

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